The work is divided into four parts, of which the first, entitled "The Everyday Life of Animals," deals with the wealth of life, the web of life, the struggle of life, the shifts for a living, the social life of animals, the domestic life of animals, and the industries of animals. The second part, on "The Powers of Life," contributed by Mr. Norman Wyld, treats of vitality, the divided labours of the body, and instinct. The third par describes "The Forms of Animal Life" and includes chapters on the life-history of animals, and their past history as read in the geological record. The fourth and last part treats of "The Evolution of Animal Life" and, besides a discussion of the influence of habits and surroundings, and of heredity, gives a sketch of the evolution of evolution theories. Appendices on the relation of animal life to human life, and on some of the best books on animal life bring the work to a conclusion.

The general arrangement of the subject-matter is, as will be seen by the above summary, well and carefully thought out, and the facts given in elucidation of the varied tendencies of organic development are skilfully marshalled and are derived from the most trustworthy sources. The information given is therefore accurate and up to date. The only suggestion we have to offer in this connection is that a little more selective elimination might have been exercised. Some facts are given in so terse and condensed a form that no one but a zoologist could appreciate their value. If a considerable number of these had been struck out and the space thus gained had been utilized in expanding those that remained, the Extensionee would have been the gainer. "The Zoological Summary of the Animal Kingdom" (pp. 210-272) might by some such process have been replaced by a sketch with more life and go in it. As it stands it will, by many readers, be gracefully skipped.

In such a work style is an important element. Here Mr. Thomson is often exceedingly happy. He has imagination and a feeling for the poetic aspect of nature. But his imagination and poetry need at times just a little chastening. When he tells us that in birds "the breathing powers are perfected and economized by a *set of balloons around the lungs*," and that their brains "are not *wrinkled with thought* like that of mammals"; when he speaks of the sponge as "*a Venice-like city of cells*"; when he describes the ciliated cells of the windpipe as "*lashed* cells," or the embryonic membranes as "*birth-robes*," and when he says that in ponds subject to drought the organism often "*sweats off a protective sheath which is not a shroud*, and waits until the rain refreshes the pools"; in these and sundry other cases of which these are samples, one may question whether the expressions which we have underlined are justified either by special elegancy or by real helpfulness to a beginner. And this we say in no spirit of hypercriticism, but as desirous of aiding the author in what is by no means an easy task.

Somewhat deeper would be our criticism of sundry expressions which are of essentially human implication and which in our opinion should not lightly be applied to animal activities. Much is said of the "love" of animals for their mates when some such phrase as "sexual appetence" would be more appropriate. For example, concerning ants we read:—"After this midsummer day's delight of love death awaits many, and sometimes most." And in the analysis of the forms of struggle for existence, we have the "struggle between rivals in love." Again, of the cuckoo it is said that, "in spite of the poets, the note of this 'blessed bird' must be regarded as suggestive of sin"! And again, "It is not quite correct to say that the cuckoo-mother is immoral because she shirks the duties of maternity; it is rather that she puts her young out to nurse because she is immoral." It is true that Mr. Thomson adds this footnote:—"The student will notice that I have occasionally used words which are not strictly accurate. I may therefore say definitely that I do not believe that we are warranted in crediting animals with moral, æsthetic, or, indeed, any conceptions." We are glad to be thus assured. But why implant notions in the text which have to be eradicated in a footnote? Does not Mr. Thomson know how easy it is to sow tares and how difficult to root them out?

Mr. Norman Wyld's chapter on "Instinct" is short, but quite to the point. We hope that he may further observe and experiment in the field of comparative psychology, for he is fully alive to the peculiar difficulties of the subject, and there is a wide field before him in which the scientific workers are none too many. In criticizing Mr. Lloyd Morgan's definition of instincts as "oft-recurring or essential to the continuance of the species," Mr. Wyld says:—"This is not quite satisfactory, for many actions that are instinctive are not oft-recurring, and many are not necessary to the preservation of the species." He does not show that there are any such actions which are neither the one nor the other. We have reason for supposing that he understood Mr. Lloyd Morgan to say that instinctive actions were "oft-recurring *and* essential to the continuance of the species." But this he did not say.

In conclusion we may repeat that "The Study of Animal Life," though by no means faultless, may be recommended to Extension students and the general reader as, in the main, accurate, readable, and suggestive.

C. LL. M.

VECTOR ALGEBRA.

Principles of the Algebra of Vectors. By A. Macfarlane, M.A., D.Sc., LL.D., F.R.S.Edin., Professor of Physics in the University of Texas. Reprint from the Proceedings of the American Association for the Advancement of Science, Vol. XL., 1891, pp. 65-117. (Salem Press, Salem, Mass., 1891).

THIS is a very suggestive contribution to the foundations of the Algebra of Vectors as recently so strongly advocated in America by Prof. Willard Gibbs, and in this country by Mr. Oliver Heaviside.

The extensive use of quaternions among physicists has been prevented by the fact that the meaning of a product of vectors has been made to depend on the use of a vector as a quadrantal versor, and by the fact that this method leads to the square of a vector being negative. The advocates of the new algebra define a product of vectors independently and in such way that the square of a vector is positive. Rotations are expressed by means of dyadics, or ratios between vectors and the quaternion notion of a vector being also a quadrantal versor is not entertained at all.

The author of this pamphlet devotes a portion of it to the consideration of quaternions, which he holds should form a distinct algebra by themselves, and he suggests a special notation for them. He restricts a quaternion proper to a *pure number* (a stretching factor) combined with a certain amount of turning. A vector, on the contrary, may be a quantity of any dimensions, possessing direction, with no suggestion of turning attached to it.

He clearly shows that the objectionable *minus* which occurs in scalar products in quaternions arises from the attempt to use the same symbol both for a quadrantal versor and for a vector, so that the laws established for dealing with one set of quantities may hold also for the other set, or for a combination of the two.

It may be worth while to notice that this minus sign of the quaternionists would disappear as an explicit symbol if they considered the second vector as being drawn from the end of the first, as AB, BC, and then took the angle ABC as being the angle between the vectors—that is to say, if, in a polygon of vectors, they were to define the angles between the successive vectors to be the *internal* angles of the polygon. Indeed, by many the internal angles of a polygon (or triangle) are considered as being *the* angles between the sides, though there is loss of real naturalness and of symmetry caused by so considering them: for instance, the connection between A, B, C and a, b, c in a spherical triangle would be greatly simplified if A, B, C were to denote the *external* angles. However, if we consider these internal angles to be the angles considered by the quaternionists, the reason for the square of a vector being negative appears at once; for if a be the quantitative part (freed from the notion of direction) of a vector $\mathbf{A}$, we have $\mathbf{A}\ \mathbf{A} = a^2 \cos 180°$, $\mathbf{A}$ and $\mathbf{A}$ being consecutive sides of the polygon which have straightened out till the internal angle between them is 180°.

It may therefore be contended that the quaternionists' *minus* is not quite irrational in vector algebra (though it cannot be said not to be inconvenient there), and that the advantage of being able to treat a vector as a quadrantal versor without having to establish a new set of formulæ far more than compensates for the loss of symmetry. On the other hand, the advocates of vector algebra without the *minus* would probably reply that they have to deal with vectors which are not in any sense the same as quadrantal or any other kind of versors, and that the imaginary completeness gained does not in any degree whatever compensate for the loss of naturalness and loss of symmetry involved in the *minus*.

The author differs from Prof. Gibbs and Mr. Heaviside in the mode in which he defines the product of two vectors, as he considers the *complete* product formed on the understanding that the multiplication shall obey the distributive law. He then finds that this complete product consists of a non-directed part, and of a directed or vector part, the former consisting of the product of the two quantities into the cosine of the angle between them, and the latter of the product of the two quantities into the sine of the same angle, having as axis the normal to the plane containing the two vectors. The angle is the angle through which the first vector (occurring on the left-hand side of the product) would have to turn to make its direction coincide with that of the second.

Prof. Gibbs and Mr. Heaviside, on the contrary, define the scalar product and the vector product as if they were entirely distinct and independent quantities. Finally the same result is attained, but Prof. Macfarlane's mode of introducing these partial products as arising naturally from applying the distributive law of multiplication would seem to have an advantage from the point of view of a student.

Prof. Macfarlane dwells emphatically on the importance of considering *dimensions* of vectors, as well as their direction, and to emphasize this he separates his vector, not into *tensor* and *unit-vector*, but into *quantity* and *direction*. Thus in the equation $\mathbf{X} = xi$, x is the quantity, and i denotes the axis. Hence the equation $jk = i$ is not a violation of dimensions, but is merely a convention as to the interpretation of a composite direction, a convention, moreover, which could only be adopted in space of three dimensions, and is the statement that the plane in which j and k lie has its orientation sufficiently indicated by the normal direction i, with the further convention that the angle from j to k shall be considered positive.

The author's notation is novel, and forms a very important feature in his treatment of the subject. The scalar product of $\mathbf{AB}$, which is $ab \cos (ab)$, he calls $\cos (\mathbf{AB})$ and the vector product he calls *Sin* $\mathbf{AB}$, its magnitude, irrespective of direction, being denoted by *sin* $\mathbf{AB}$. Possibly an improvement in this latter would be to denote it by *sin ab*, and then the capital letter in the complete vector would become unnecessary.

The particular symbol used to denote a scalar or a vector product is a matter of secondary importance, but is a matter which must sooner or later be settled if vector-algebra is to come into general use. Lord Kelvin is of opinion that a function-symbol should be written with not less than three letters, and Prof. Macfarlane's notation obeys that law, and is moreover easy to work with, but is incomplete, being applicable to products of two vectors only. Mr. Heaviside uses no prefix at all to a scalar product, but considers that $\mathbf{AB}$ means the scalar product. He uses the quaternionic expression $V\,\mathbf{AB}$ for the vector product. Prof. Gibbs uses no prefix for either, but denotes the scalar product by $\mathbf{A} \cdot \mathbf{B}$, and the vector product by $\mathbf{A} \times \mathbf{B}$. The three-lettered prefix seems the clearest in both cases to denote the special product intended, and the symbols *cos* and *sin* are more or less suggestive.

In forming a product of three vectors, Prof. Macfarlane makes the convention that $\mathbf{ABC}$ shall mean $(\mathbf{AB})\mathbf{C}$, the combination commencing on the left. In his notation this product expands into

$$\begin{aligned} &(\cos \mathbf{AB} + \operatorname{Sin} \mathbf{AB})\mathbf{C} \\ &= \cos (\cos \mathbf{AB} \,.\, \mathbf{C} + \operatorname{Sin} \mathbf{AB} \,.\, \mathbf{C}) + \operatorname{Sin} (\cos \mathbf{AB} \,.\, \mathbf{C} + \operatorname{Sin} \mathbf{AB} \,.\, \mathbf{C}) \\ &= \cos (\operatorname{Sin} \mathbf{AB} \,.\, \mathbf{C}) + \operatorname{Sin} (\cos \mathbf{AB} \,.\, \mathbf{C}) + \operatorname{Sin} (\operatorname{Sin} \mathbf{AB} \,.\, \mathbf{C}) \\ &= \operatorname{vol} \mathbf{ABC} + \mathbf{C} \,.\, \cos \mathbf{AB} + \operatorname{Sin} (\operatorname{Sin} \mathbf{AB} \,.\, \mathbf{C}) \end{aligned}$$

which finally becomes

$$= \operatorname{vol} \mathbf{ABC} + \mathbf{C} \cos \mathbf{AB} + \mathbf{B} \cos \mathbf{AC} - \mathbf{A} \cos \mathbf{BC} \,;$$

where vol (**ABC**) denotes the volume of the parallelopiped of which **ABC** are three adjacent edges. The only objection to this name lies in its suggesting that **A**, **B**, **C** are *linear* vectors.

Here appears the defect in the author's *cos* and *sin* notation, in that it cannot be applied to the products of three vectors, or at least that the special reason for its use has disappeared, and the author does not suggest so applying it.

But there is a certain perspicuity attained by this very limitation of the *cos* and *sin* notation to the products of only two vectors, inasmuch as there can be no ambiguity in the meaning of an expression in which they occur, even if brackets are omitted or placed differently. Indeed, instead of cos (Sin **AB** . **C**) the author writes cos (Sin **AB**)**C**, which seems a curious use of the bracket. But cos Sin **AB** . **C**, or preferably cos **C** Sin **AB**, is just as explicit, and even cos Sin **ABC**, though wrong to write as being puzzling, can only have the same meaning.

The author concludes with short sections on dyads and matrices, on scalar- and vector-differentiation, including scalar-differentiation of a quaternion. On the last page are a series of propositions relating to the addition of scalar and vector quantities situate at, or passing through, specified points.

The pamphlet is confined solely to statements of principles and the section devoted to dyads and matrices is very condensed, so that it is not in any sense a text-book for students. It is rather a synopsis of the subject, with the introduction of a special notation which the author has found useful. A text-book of vector algebra, with examples showing its application to problems in geometry, mechanics, and general physics, and contrasting the method with the Cartesian method of treating the same problems, is much needed, as many physicists are becoming interested in the new algebra, owing in great measure to Mr. O. Heaviside's able exposition of its principles and applications in the *Electrician* and elsewhere.

THE LAKE OF GENEVA.

Le Léman: Monographie Limnologique. F. A. Forel. Tome Premier. (Lausanne: F. Rouge, 189–.)

PROF. FOREL has been for some years occupied in studying the Lake of Geneva, and has now published the first instalment of the fruits of his labours. The work, when finished, is intended to be a complete monograph of the history of a single lake, and will be a most important contribution to an interesting branch of physical geography. In the present volume the geography, the hydrography, the geology, the climatology, and the hydrology of Lake Léman are discussed, after some introductory matter relating to the instruments employed in sounding with other preliminaries. But, though only a single volume, the work embraces so many questions that we must, for want of space, confine our notice mainly to one, which, of late years, has attracted the most attention, at any rate in this country, viz. What has been the origin of the lake basin? Was it formed by the old Rhone glacier or in some other way? The especial value of Prof. Forel's memoir is the number of new facts which it brings to bear on the problem thus propounded.

The Lake of Geneva, however it may have been caused, is more modern than the middle of the Miocene period: "Le lac n'existait pas encore, la vallée du Léman n'était pas même indiquée quand la mer helvétienne déposait les mollasses d'Epalinges et du Mont." Its slopes, and almost certainly its bed, are covered with glacial deposits, of later date than the formation of its basin. Terraces around its shore indicate that its waters once reached a higher level, the greatest elevation which can be identified with certainty, being about 30m. above the present surface. The next pause was at 10m.; after that the lake sank (the fall always being rapid) to its present level. Traces of still higher terraces are to be found on the north shore, but as these neither can be identified on the opposite side, nor correspond with any natural barrier in the course of the Rhone below the lake. Prof. Forel doubts whether they indicate old levels of its waters.

Lake Léman consists of two basins. The first and larger extends from the embouchure of the Rhone to the narrow of Promenthoux. At the east end the slope of the cone of alluvium deposited by the Rhone in no part exceeds 25°. First comes a zone of very shallow water off the actual shore line; to this succeeds a more rapid slope, which gradually eases off as it descends. The current of the Rhone has made and maintains a well-marked channel in this mass of detritus, and the contour lines are affected down to 250m. At the embouchure of the Dranse, on the south shore, another alluvial cone has been deposited. This, however, is rather steeper, but it is much smaller, and does not perceptibly affect the course of the subaqueous contour lines below about 200m. On the north side of the basin the slope varies. Under the walls of Chillon the descent is rapid, amounting to 137 in 100; it is nearly the same near St. Gingolph on the opposite shore, doubtless indicating submerged crags; but it is generally more moderate. West of Vevay it is about one in four, whence it changes gradually to one in ten opposite to Ouchy.

West of this port the descent is still more gentle, and so it continues round the western end of the basin, the lip of the latter being 75m. below the surface. The contours of the south side correspond generally with those of the north, and the form of the basin is evidently related to the geology of the district, being narrower and steeper among the harder rocks at the eastern end. The deepest part is a large rudely triangular area, the apex pointing towards the west, and the base lying roughly north and south, extending from almost opposite to the embouchure of the Dranse to near Lutry. All this area is an almost level plain, for it is wholly below the 300m. contour line, but the greatest depth obtained was only 309·7m.

The Petit Lac may be described as a comparatively narrow and shallow trough, rising very slowly from a depth of about 70 to 50 metres, and then gradually mounting to the embouchure of the Rhone, its bed being slightly interrupted by five small shallow basins, which roughly speaking, have a linear arrangement, but their floors only sink four or six yards at most below the general level.

The lake to some extent is still held up by the huge mass of gravel brought down by the Arve, through which the two rivers have now cut their channels on either side of the plateau of La Bâtie below Geneva. But it is